Some applications of the Biham-Chen attack

Conference Hash Functions

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Overview of the talk

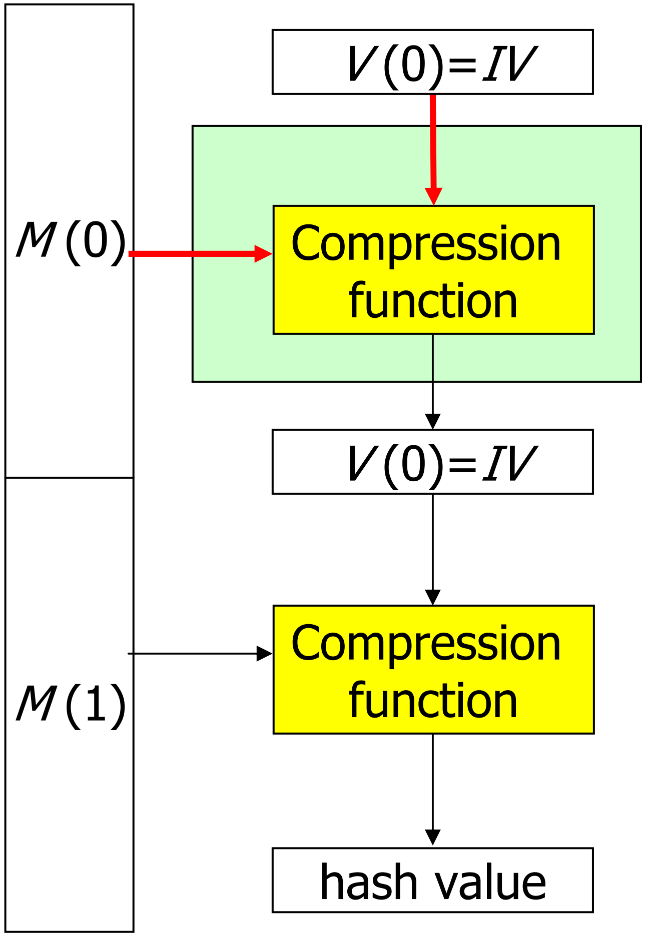
- Introduction to some resistances of hash functions
- Cryptanalysis of hash functions in encryption mode.
- Description of the Biham-Chen attack
- Applications to MD5 and a SHA-256 variant
- Conclusions

Some resistances of hash functions

- Near-collision resistance
  - Resistance against attacks finding a pair of hash values which differ in only small number of bit positions.
- Pseudo-collision resistance
  - Resistance against collision attacks where different initial vectors can be chosen.
- Randomness
  - Resistance against attacks distinguishing it from a random function.

Pseudo-collision resistance

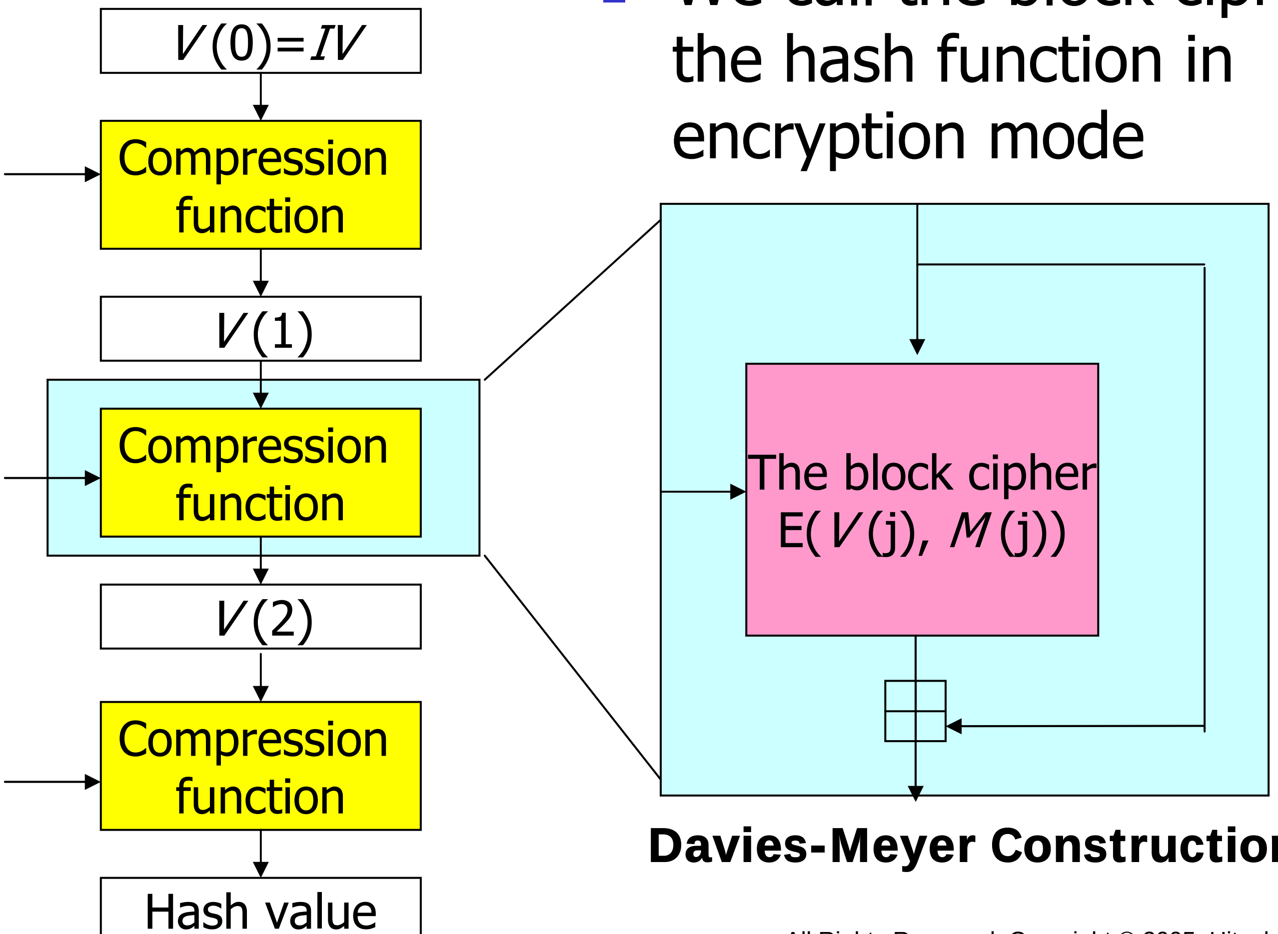
The MD-construction



- Resistance when 2 inputs controlled.
- Important in the theory of the MD-construction
- There could be some application which requires the underlying hash function to have this resistance
  - Knudsen *et al*, Preimage and pseudo-collision attack on MD2, FSE2005

Hash Function in encryption mode

- We call the block cipher *E* the hash function in encryption mode



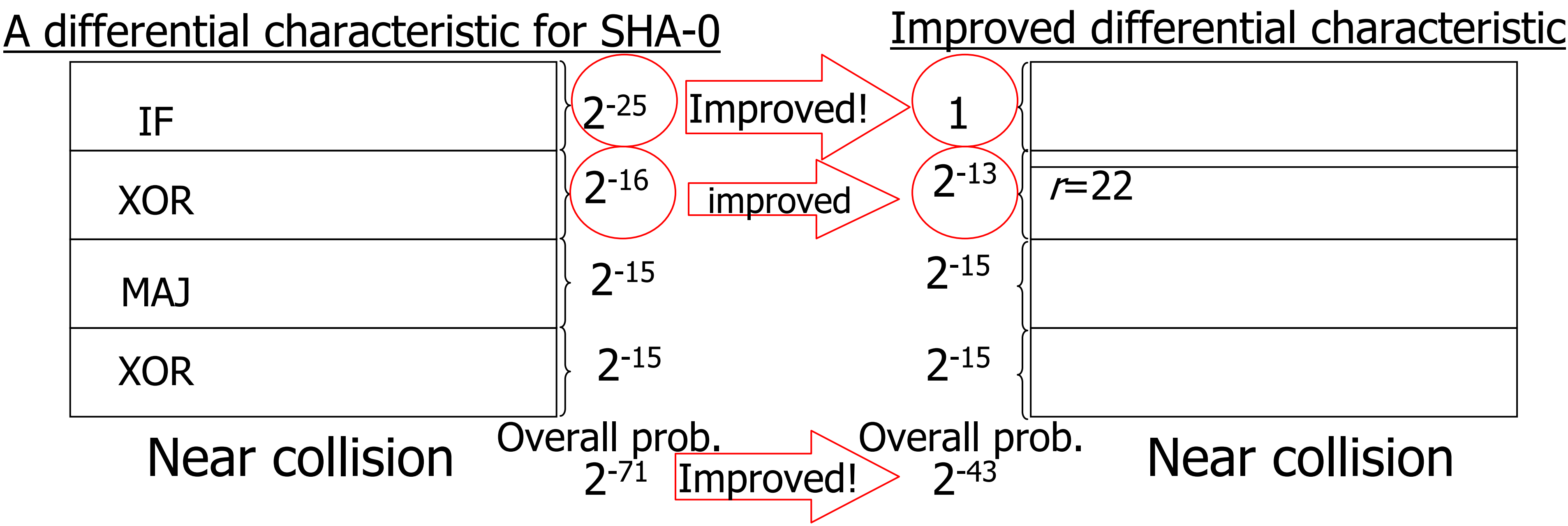
Davies-Meyer Construction

Cryptanalysis of Hash functions in Encryption Mode

- Differential cryptanalysis of SHA-1
  - Handschuh *et al.*, SHACAL, Submission to the NESSIE project, 2000.
- Slide attack on SHA-1
  - Saarinen, Cryptanalysis of Block Ciphers Based on SHA-1 and MD5, FSE2003.
- Attack on MD5 which finds one high-probability differential characteristic.
  - Saarinen, Cryptanalysis of Block Ciphers Based on SHA-1 and MD5, FSE2003.
- Attack which distinguishes HAVAL from a random function.
  - Yoshida *et al.*, Non-randomness of the Full 4 and 5-pass HAVAL, SCN2004.

Description of Biham and Chen attack

- Near-collision attack on SHA-0
  - Biham and Chen, near-collision of SHA-0, CRYPTO 2004



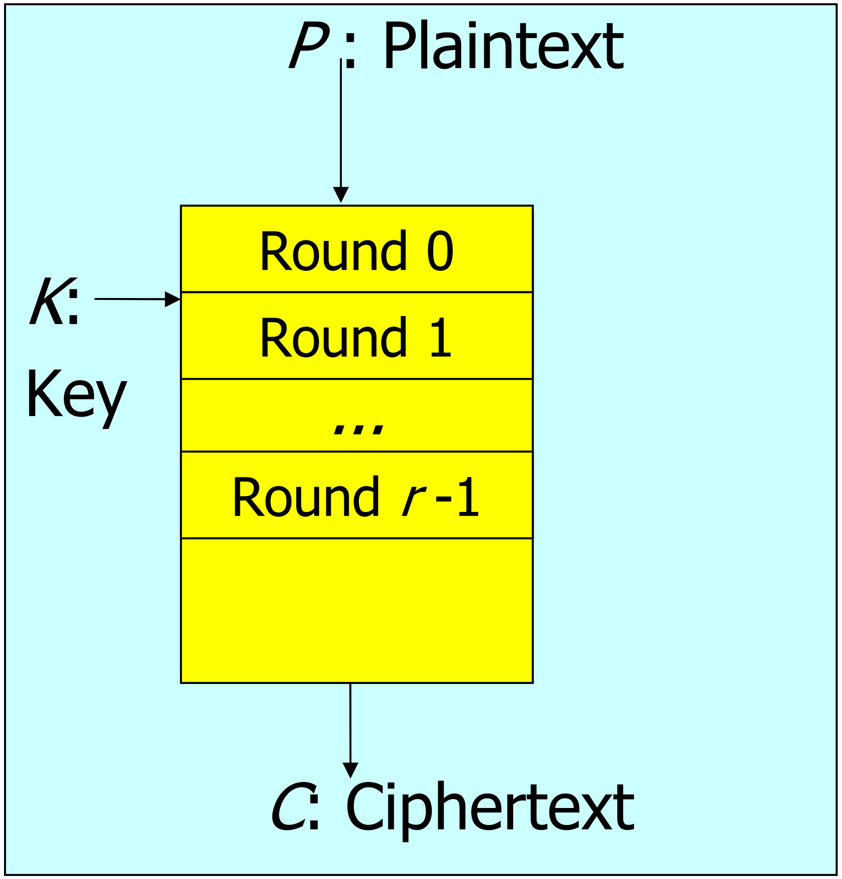
- Start the collision search from some intermediate round *r*.
- Use messages generated from *neutral bits*

Concept of neutral bits

- *Neutral bits* do not affect the difference for r rounds.
- Obtaining $k(r)$ neutral bits allows to generate a set of $2^{k(r)}$ messages.
- Using this set gives us a better probability for r rounds than the probability when using a set of randomly chosen messages.

Differential Cryptanalysis of a Hash Function in Encryption Mode

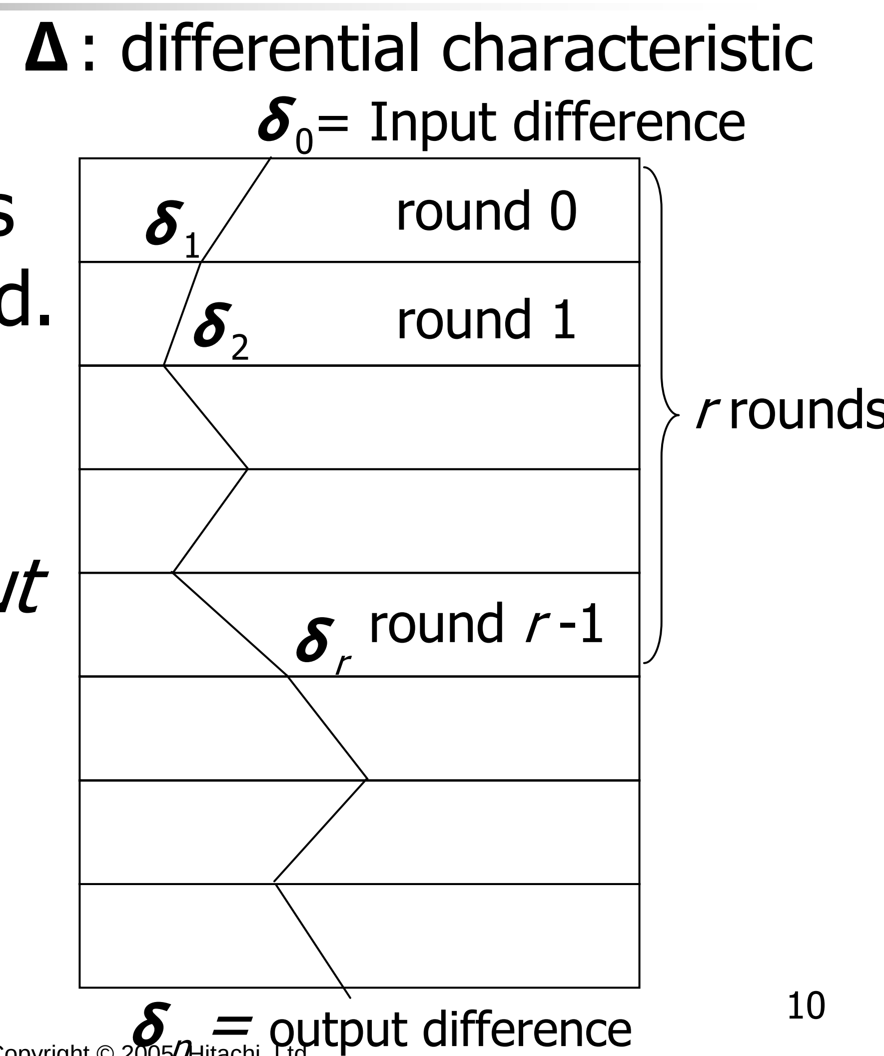
- We assume:
  - A differential characteristic, Δ has already been found
  - the key value K is fixed to one value $K=K_0$



$E(K, P)$

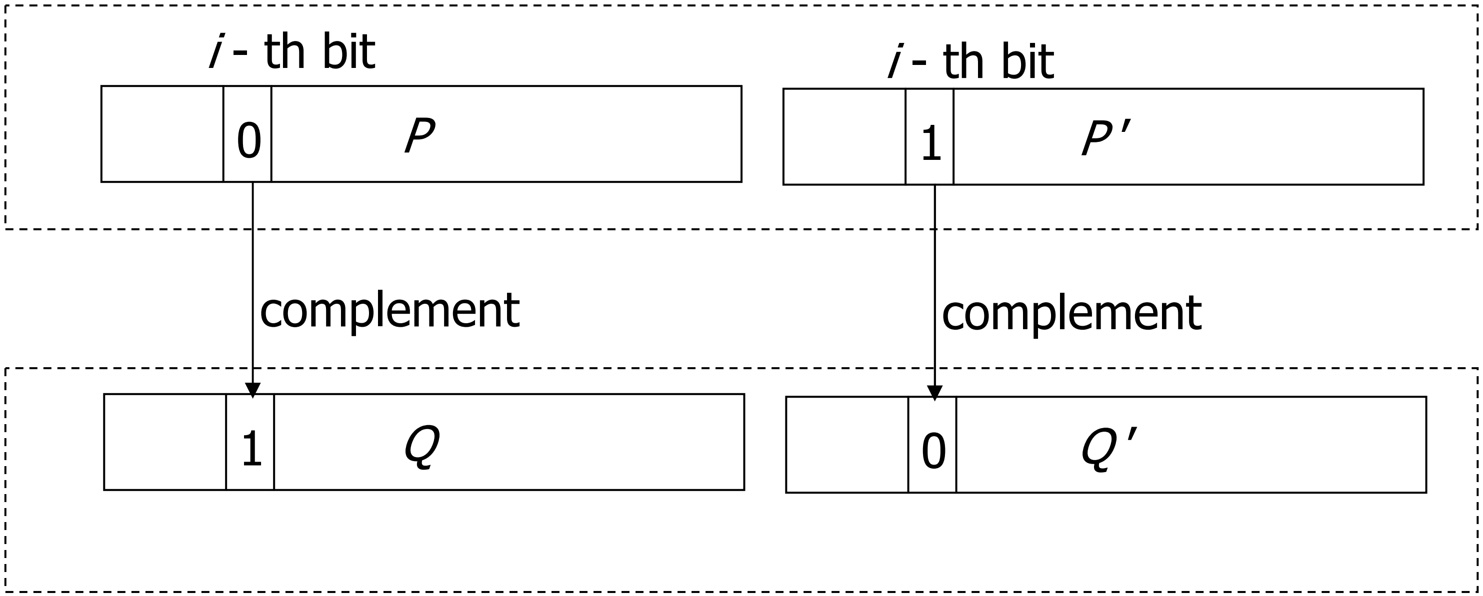
Differential Cryptanalysis of a Hash Function in Encryption Mode

- *Δ* defines the expected differences δ_r of the values of all registers in each round.
- Definition.
  (P, P') conforms to Δ_r if the differences at the output of the first r rounds are as expected.



Differential Cryptanalysis of a Hash Function in Encryption Mode

Assume that (P, P') conforms to Δ_r

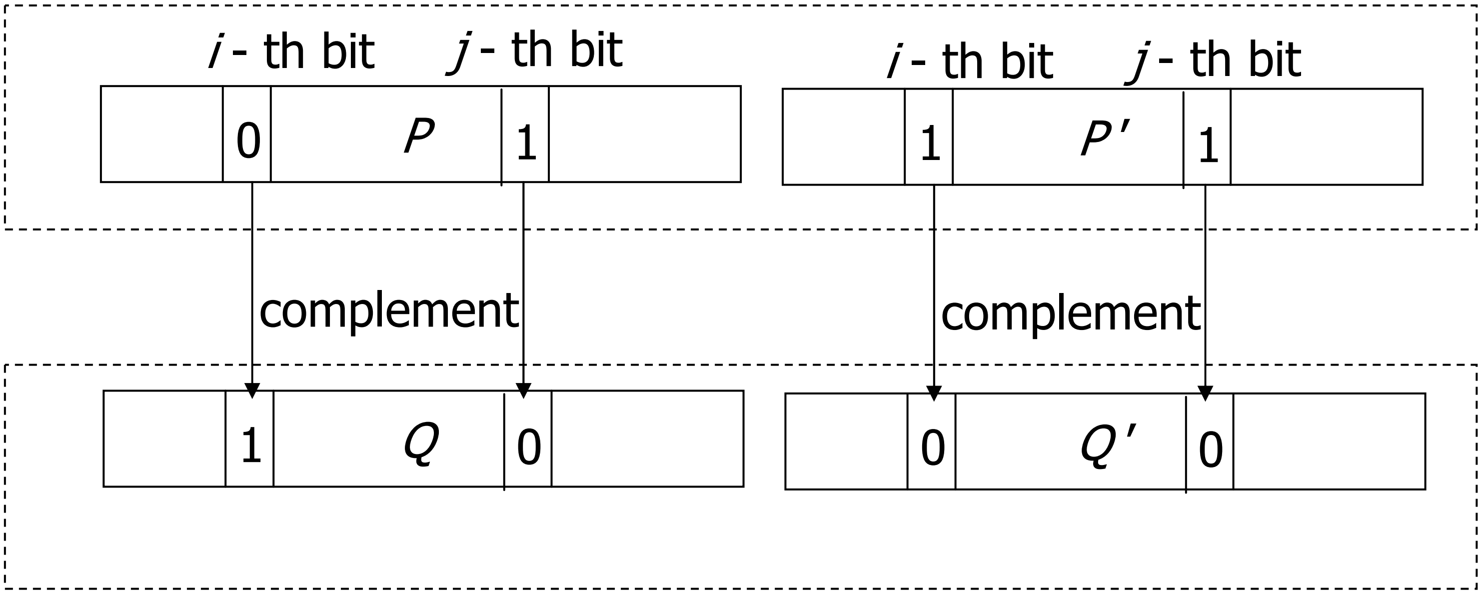


If (Q, Q') conforms to Δ_r, the *i* -th bit is called neutral bit.

Differential Cryptanalysis of a Hash Function in Encryption Mode

Assume that *(P, P′)* conforms to Δ_r

Let *i* -th bit and *j* -th bit be neutral bits.



If *(Q, Q')* conforms to Δ_r , there is an edge between *i* -th bit and *j* -th bit.

An algorithm for finding a 2-neutral set

Step1: Find a pair of plaintexts that conforms to Δ_r for some r

Step2: Find the set S of singles of neutral bits

Step3: Find neutral pairs in S

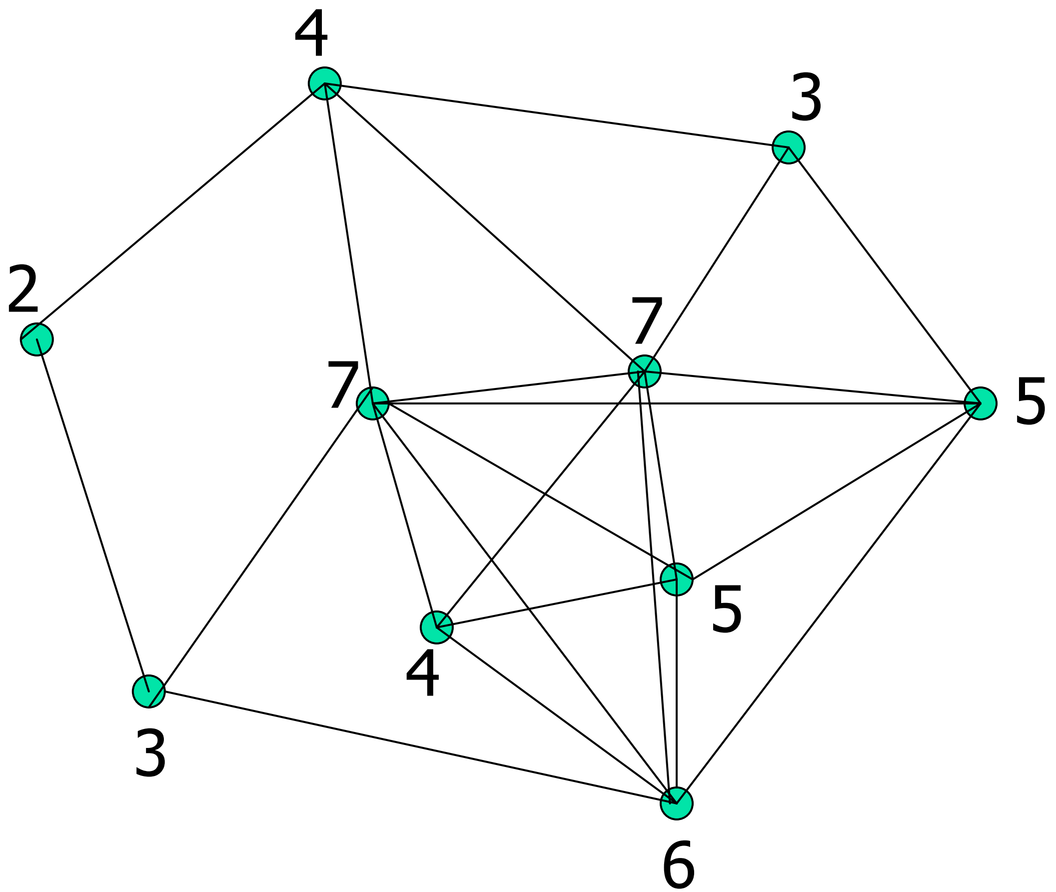
Step4: Count the number of edges for each element of S

Step5: If the resulting set is a neutral set, break

Otherwise remove from S one of the elements which has the least number of edges.

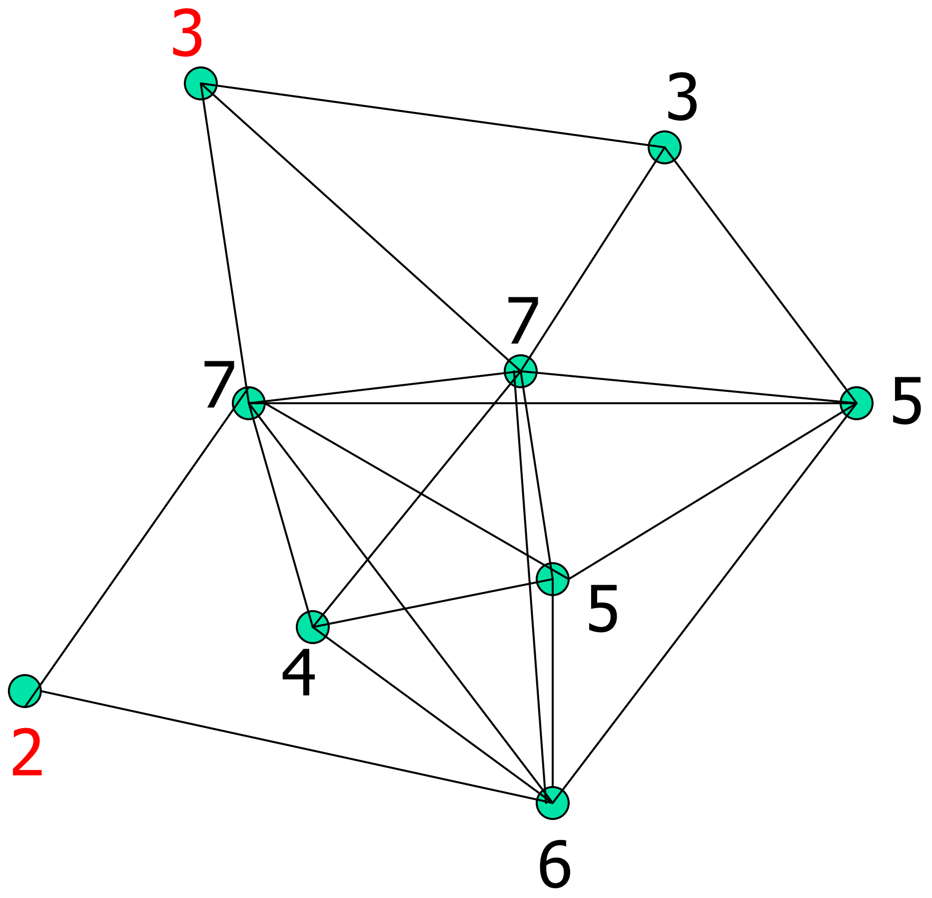
Let the resulting set be S go to step 3

How to construct 2-neutral sets



- *neutral bits* do not affect the difference for *r* rounds.

How to construct 2-neutral sets



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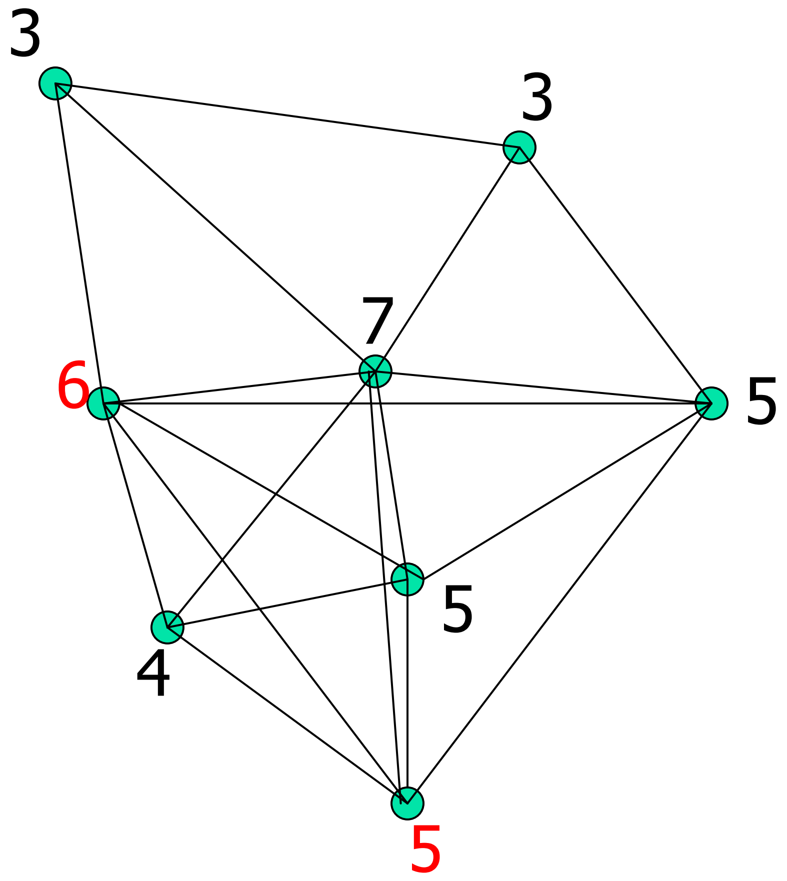

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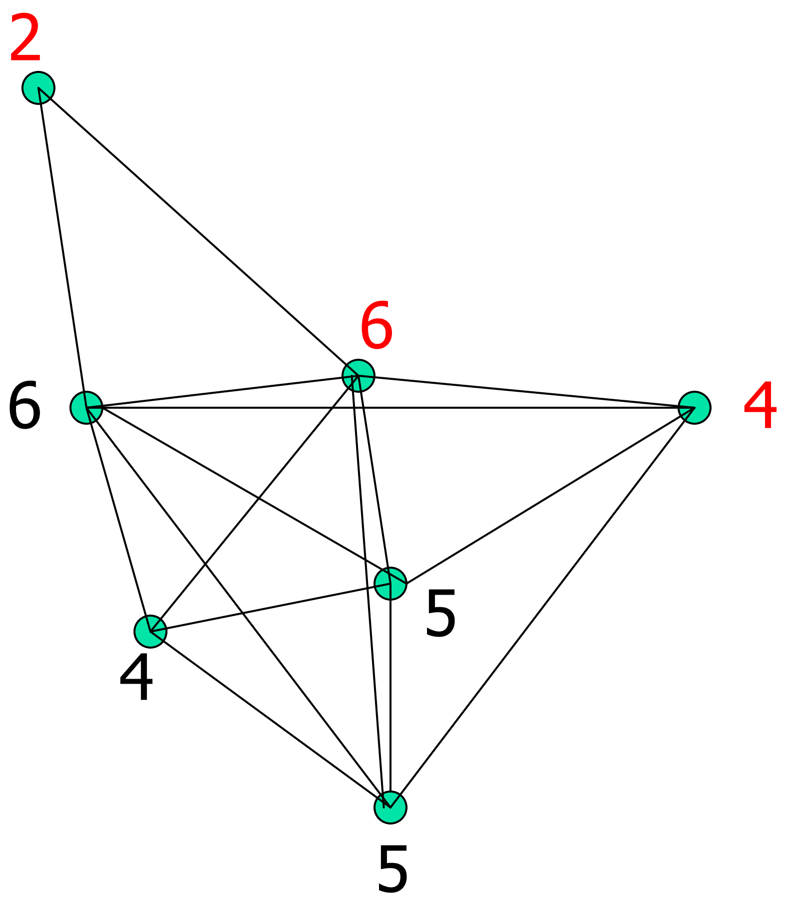

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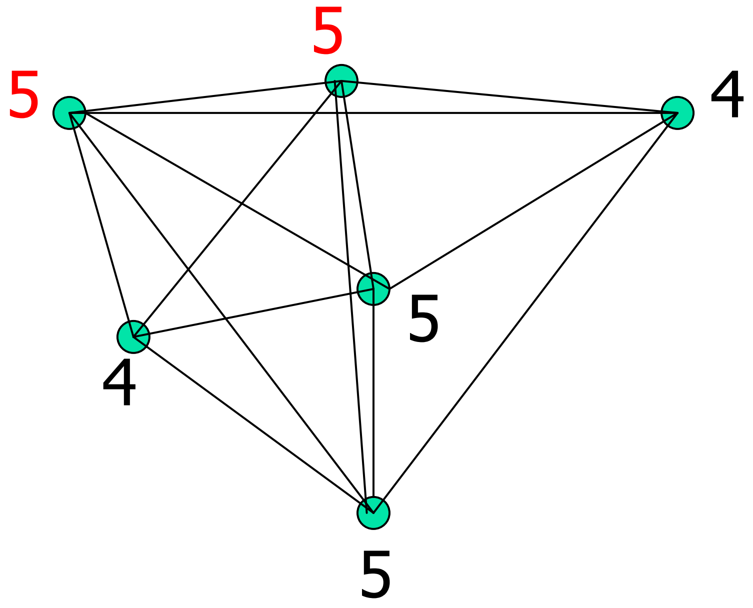
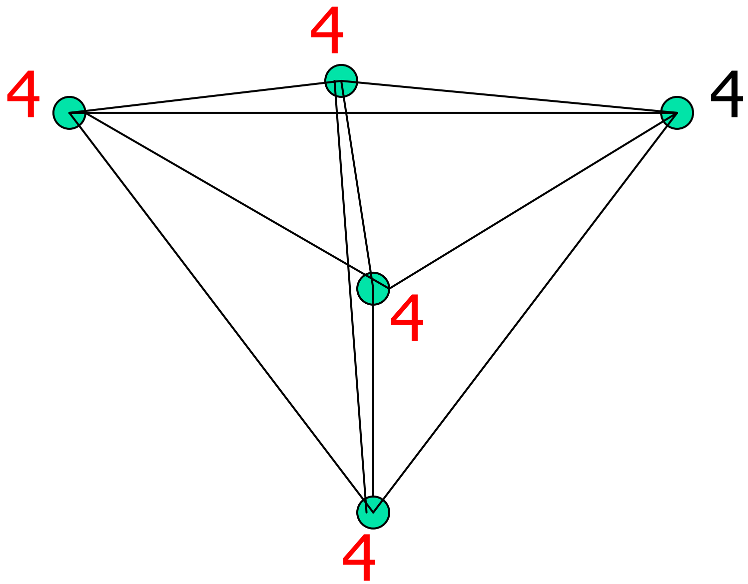
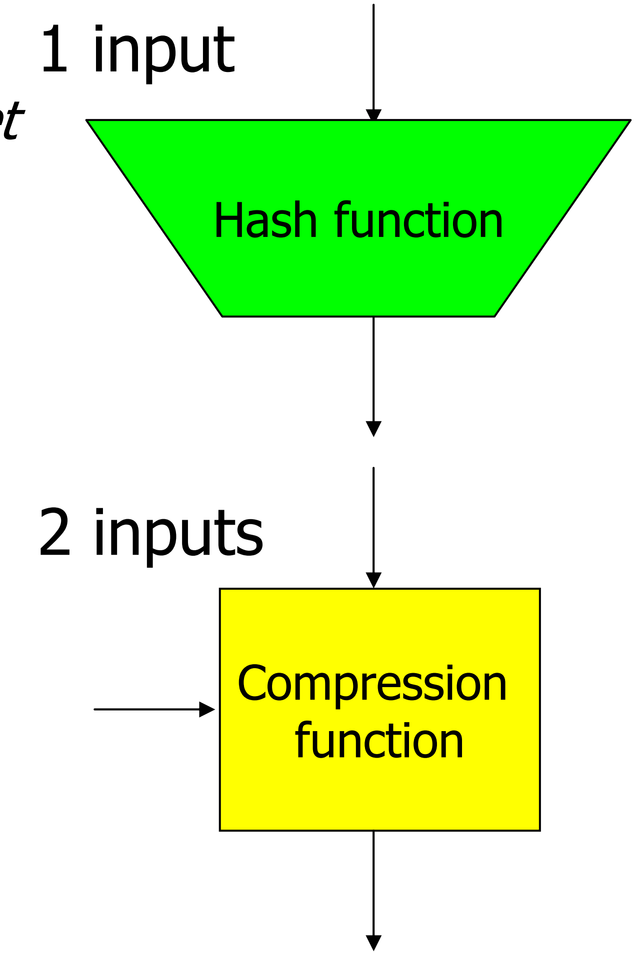


How to construct 2-neutral sets

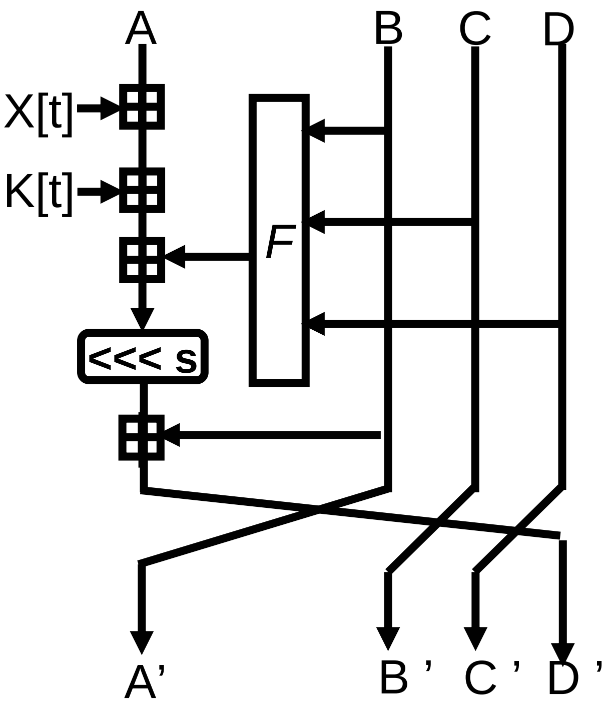


Application to MD5 hash function

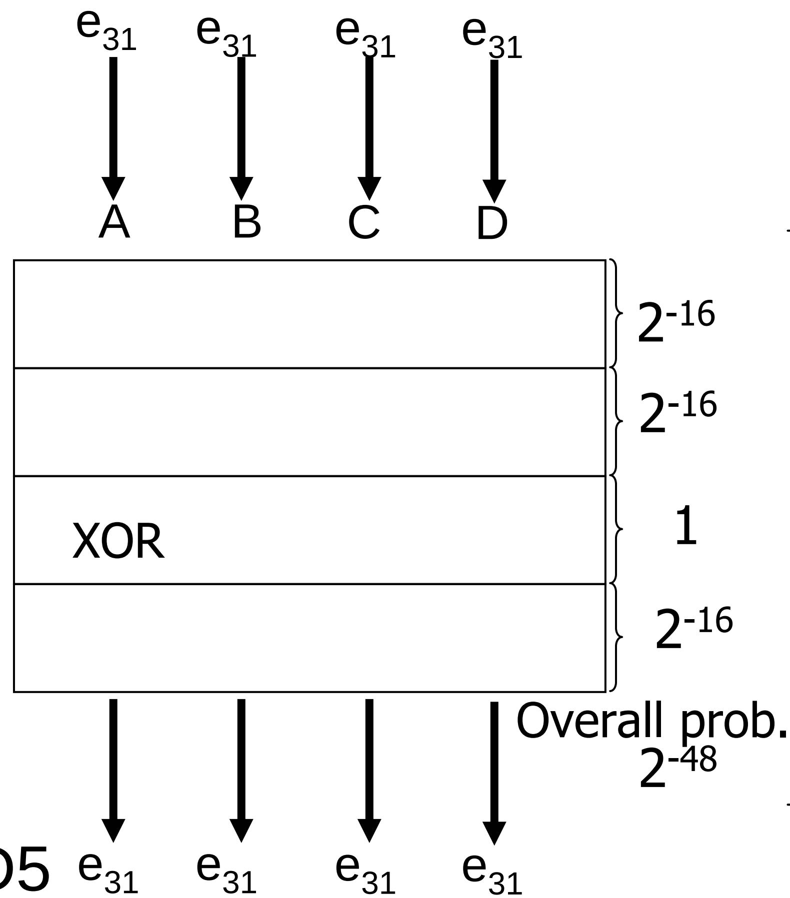
- Attacks on MD5
  - Attacks for finding collisions (Wang *et al.*, Eurocrypt 2005).
- Attacks on the compression function of MD5
  - Attacks for finding pseudo-collisions (Dobbertin. Cryptanalysis of MD5 Compress., at Eurocrypt '96 rump session)
  - Attacks for finding pseudo-collisions (Saarinen, FSE 2003)

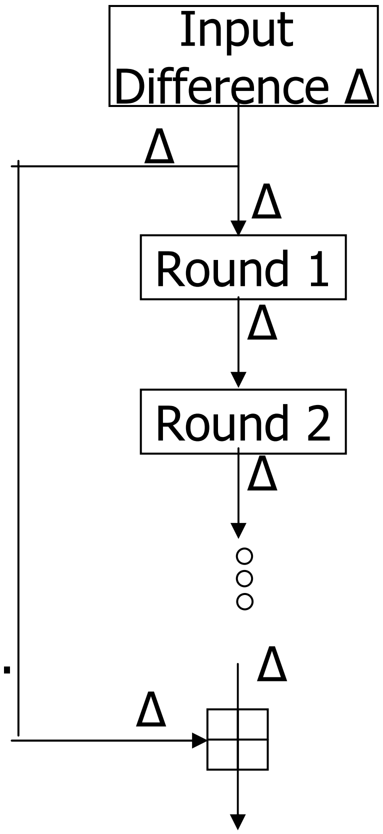


Saarinen’s iterative characteristic



The step function of MD5





Pseudo-collision

Experimental results

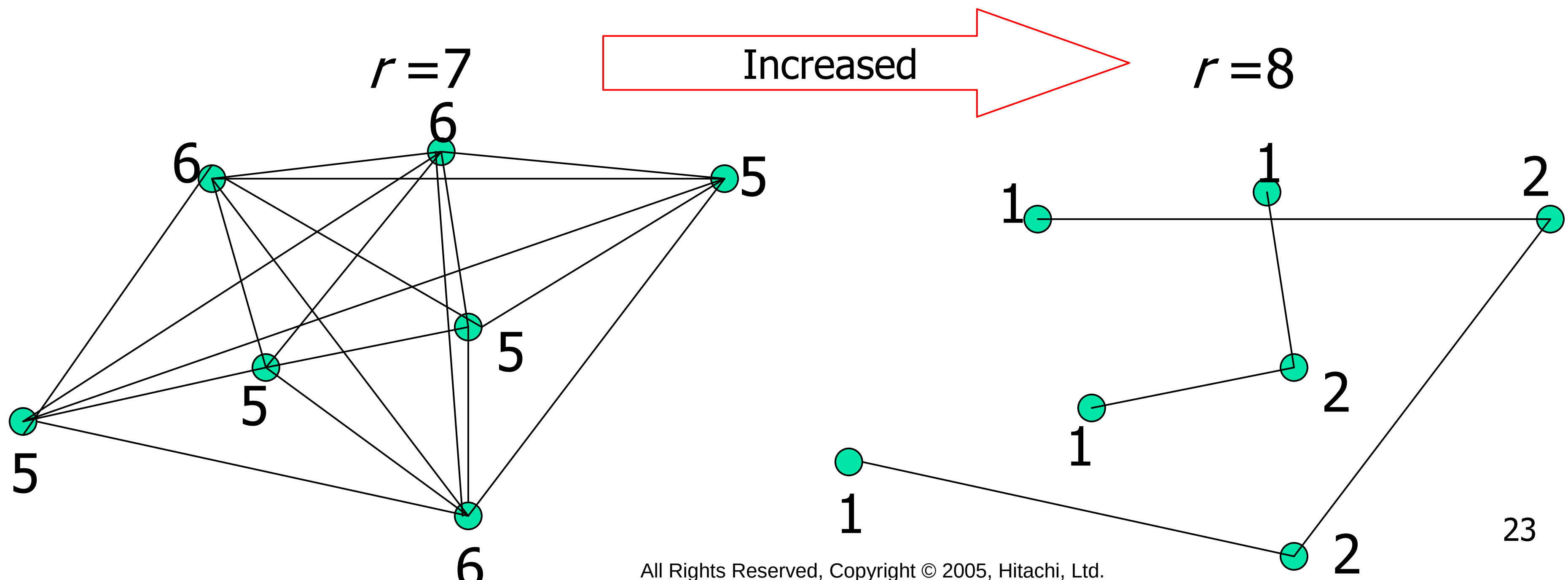
- 4 different non-linear functions are used.
- It is interesting to see the improvements for each of the 16 rounds.

Probability comparison

Rounds	Previous probability	Improved probability
0-15	2^{-16}	$2^{-6.46}$
16-31	2^{-16}	$2^{-9.33}$
32-47	1	1
48-63	2^{-16}	$2^{-7.22}$

Observations

- The obtained sets could be too small to attack many rounds.
- When *r* is increased, the number of edges for each element is rapidly decreased.



Experimental results

A set of neutral bits of size 34 for $r = 7$, which is almost 2-neutral
(The bits are numbered in the range 0, ..., 127)

P = 0x938858dc 0xf310b6b4 0xa9f02359 0x1207a9e3
P'= 0x138858dc 0x7310b6b4 0x29f02359 0x9207a9e3
S = {3,4,6,8,9,13,20,21,22,30,33,34,40,41,43,47,57,58,59, 62,65,66,74,88,96,104,105,106,107,108,113,123,125,126}

Experimental results

- Probability of the 49-round characteristic obtained from *S* is about 2^{-27}, which is 2^{6} higher than the original probability.
- In practice, we found 132 pseudo collisions for 49 rounds of MD5 with complexity 2^{34}

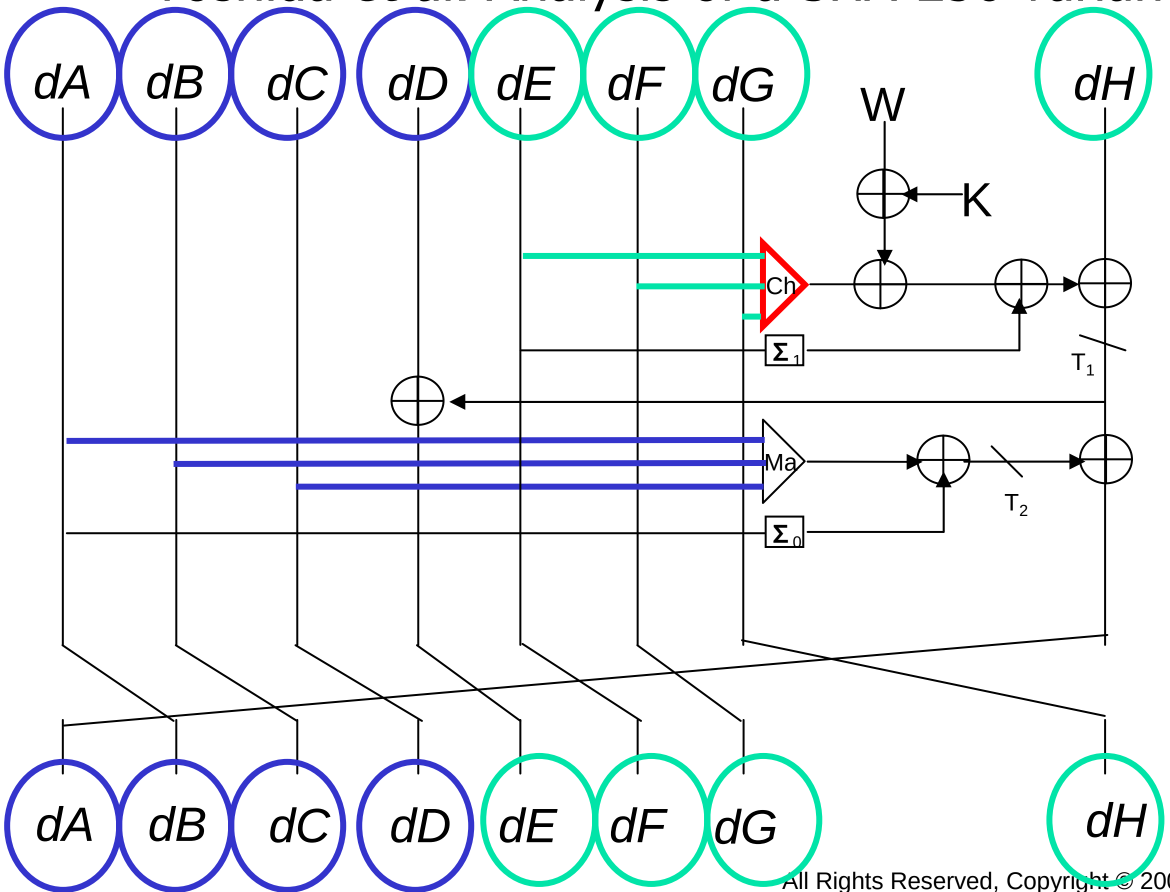
Plaintext pair which produces a pseudo collision for 49 rounds:

Q = 0xd3b8788c 0xf910b4b6 0xa9f02359 0x1a05b4e3
Q'= 0x53b8788c 0x7910b4b6 0x29f02359 0x9a05b4e3

Application to a SHA-256 variant

One-round iterative for a SHA-256 variant (SHA-2-XOR) presented

- Yoshida *et al*. Analysis of a SHA-256 variant., at FSE 2005 rump session



- Best probability 2^{-8}
- Pseudo-collision for 4-round SHA-2-XOR with complexity 2^{32}

Apply our technique here

- Pseudo-collision for 8-round with same complexity

Conlusions

- We discussed some resistances and tried to apply the Biham-Chen attack to study hash functions regarding these resistances.
- Some improved results on MD5 and a SHA-256 variant were presented.
- The generic approach here may find interesting results on hash functions regarding these two resistances for which differential characteristics have been already found.